Determinacy after Frege

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Frege's requirement of the determinacy of sense

The distinction between symbols that can be taken to mean various things and those that have a fully determinate sense in *Begriffschrift* (1879)

"Accordingly, I divide all the symbols I use into *those that can be taken to mean various things* and *those that have a fully determinate sense.* The first kind are *letters,* and their main task is to be the expression of *generality.* For all their indeterminateness, it must be laid down that a letter *retains* in a given contact the meaning once given to it."
(Frege, Begriffschrift, §1, 1, in *Translation from the Philosophical Writings of Gottlob Frege*).

"Alle Zeichen, die ich anwende, theile ich daher ein *in solche, unter denen man sich Vershiedenes vorstellen kann,* und *in solche die einen ganz bestimmten Sinn haben."*
(Frege, *Begriffschrift und andere Ausätze, Zweite Auflage*, 1)

The requirement of the sharp delimitation of concepts in *Function and Concept* (1891)

"This involves the requirement as regards concepts, that, for any argument, they shall have a truth-value as their value; that it shall be determinate, for any object, whether it falls under the concept or not. In other words: as regards concepts we have a requirement of sharp delimitation; if this were not satisfied it would be impossible to set forth logical laws about them. For any argument x for which 'x + I' were devoid of reference, the function x + 1 = 10 would likewise have no value, and thus no truth-value either, so that the concept: 'what gives the result 10 when increased by 1' would

have no sharp boundaries. The requirement of the sharp delimitation of concepts thus carries along with it this requirement for functions in general that they must have a value for every argument."
(Frege, Function and Concept, p. 33 in *Translation from the Philosophical Writings of Gottlob Frege*).

An empty concept is not an indeterminate concept: it must be determinate for every object whether it falls under a concept or not; a concept word which does not meet this requirement on its meaning is meaningless (1892-1895)

"If it is a question of the truth of something—and truth is the goal of logic—we also have to inquire after meanings; we have to throw aside proper names that do not designate or name an object, though they may have a sense; we have to throw aside concept-words that do not have a meaning. **These are not such as, say, contain a contradiction—for there is nothing at all wrong in a concept's being empty—but such as have vague boundaries. It must be determinate for every object whether it falls under a concept or not; a concept word which does not meet this requirement on its meaning is meaningless.**"
(Frege "*Comments on Sense and Meaning"*, 1892-1895, p. 122, in *Frege, Posthumous Writings*, 1979)

Logic must demand of concept-words that the step from the word to the sense and from the sense to the meaning be determinate beyond doubt (1892-1895)

"Logic must demand not only of proper names but of concept-words as well that the step from the word to the sense and from the sense to the meaning be determinate beyond any doubt. Otherwise we should not be entitled to speak of a meaning at all. Of course this holds for all signs and combinations of signs with the same function as proper names or concept-words".
(Frege "*Comments on Sense and Meaning"*, 1892-1895, p. 125, in *Frege, Posthumous Writings*, 1979)

Frege to Peano: Logic can only recognize sharply delimited concepts (1896)

(…) But logic can only recognize sharply delimited concepts. Only under this presupposition can it set up precise laws. The logical law that there is no third case besides
a is *b*
and
a is not *b*
is really only another way of expressing our requirement that a concept *(b)* must be sharply delimited. The fallacy known by the name of 'Acervus' rests on this, that words like 'heap' are treated as if they designated a sharply delimited concept whereas this is not the case. Just as it would be impossible for geometry to set up precise laws if it tried to recognize threads as lines and knots in threads as points, so logic must demand sharp limits of what it will recognize as a concept unless it wants to renounce all precision and certainty. Thus a sign for a concept whose content does not satisfy this requirement is to be regarded as meaningless from the logical point of view. It can be objected that such words are used thousands of times in the language of life. **Yes; but our vernacular languages are also not made for conducting proofs. And it is precisely the defects that spring for this that have been my main reason for setting up a conceptual notation.** (...)

The situation is quite similar for relations. A conditional definition of a sign for a relation, as for example for identity, decides only in some cases, not in all, whether the relation holds. Thus your definition I, sect. 4, 2 decides whether *a* is identical with *b* only in the case where *a* and *b* are classes; it does not therefore give the sign of identity a meaning independent of *a* and *b*; i.e., it does not give it a meaning at all. Besides the two cases
a is identical with *b*
and
a is not identical with *b*
there still remains a third case here, that of undecidability, whereas logic does not tolerate a third case."
(Frege to Peano, 29.09.1896, p.114-115 in *The Philosophical and Mathematical Correspondence)*

The law of excluded middle as the requisite that the concept should have a sharp boundary (1903)

A definition of a concept (of a possible predicate) must be complete; it must unambiguously determine, as regards any object, whether or not it falls under the concept (whether or not the predicate is truly assertible of it). Thus there must not be any object as regards which the definition leaves in doubt whether it falls under the concept; though for us men, with our defective knowledge, the question may not always be decidable. We may express this metaphorically as follows: the concept must have a sharp boundary. If we represent concepts in extension by areas on a place, this is admittedly a picture that may be used only with caution but here it can do us good service. To a concept without sharp boundary there would correspond an area that had not a sharp boundary-line all round, but in places just vaguely faded away into the background. This would not really be an area at all; and likewise a concept that is not sharply defined is wrongly termed a concept. Such quasi-conceptual constructions cannot be recognized as concepts by logic; it is impossible to lay down precise laws for them.
(Frege, *Grundgestze der Arithmetik,* Vol. ii, §56 *Principles of Definition*. I. *Principle of Completeness,* p.159 in *Translation from the Philosophical Writings of Gottlob Frege*).

Logic can recognize a relation only if it is determinate (1903)

"We get the same case for a relation as for a concept: logic can recognize a relation only if it is determinate, as regards anyone object and any other object, whether or not the one stands to the other in that relation. Here too we have a *tertium non datur*; the case of its being undecided is ruled out."
(Frege, *Grundgestze der Arithmetik,* Vol. ii, Section 62 *Principle of Completeness,* p.165 in *Translation from the Philosophical Writings of Gottlob Frege*).

Wittgenstein's requirement of the determinacy of sense

The requirement of the determinacy of sense in the *Tractatus*

3. Das logische Bild der Tatsachen ist der Gedanke.

3.1 Im Satz drückt sich der Gedanke sinnlich wahrnehmbar aus.

3.2 Im Satze kann der Gedanke so ausgedrückt sein, dass den Gegenständen des Gedankens Elemente des Satzzeichens entsprechen.

3.201 Diese Elemente nenne ich „einfache Zeichen“ und den Satz „vollständig analysiert“.

3.23 Die Forderung der Möglichkeit der einfachen Zeichen ist die Forderung der Bestimmtheit des Sinnes.

3.24 Der Satz, welcher vom Komplex handelt, steht in interner Beziehung zum Satze, der von dessen Bestandteil handelt.

Der Komplex kann nur durch seine Beschreibung gegeben sein, und diese wird stimmen oder nicht stimmen. Der Satz, in welchem von einem Komplex die Rede ist, wird, wenn dieser nicht existiert, nicht unsinnig, sondern einfach falsch sein.

Dass ein Satzelement einen Komplex bezeichnet, kann man aus einer Unbestimmtheit in den Sätzen sehen, worin es vorkommt. Wir *wissen*, durch diesen Satz ist noch nicht alles bestimmt. (Die Allgemeinheitsbezeichnung *enthält* ja ein Urbild.)

Die Zusammenfassung des Symbols eines Komplexes in ein einfaches Symbol kann durch eine Definition ausgedrückt werden.

3.25 Es gibt eine und nur eine vollständige Analyse des Satzes.

3.26 Der Name ist durch keine Definition weiter zu zergliedern: er ist ein Urzeichen.

3.3 Nur der Satz hat Sinn; nur im Zusammenhang des Satzes hat ein Name Bedeutung.

3.23 The postulate of the possibility of the simple signs is the postulate of the determinateness of the sense. (Ogden)

3.23 The requirement that simple signs be possible is the requirement that sense be determinate. (Pears/McGuinness)

3.24 That a propositional element signifies a complex can be seen from an indeterminateness in the propositions in which it occurs. We *know* that everything is not yet determined by this proposition. (The notation for generality *contains* a prototype.)

The combination of the symbols of a complex in a simple symbol can be expressed by a definition.

(Ogden)

3.24 When a propositional element signifies a complex, this can be seen from an indeterminateness in the propositions in which it occurs. In such cases we *know* that the proposition leaves something undetermined. (In fact the notation for generality *contains* a prototype.)

The contraction of a symbol for a complex into a simple symbol can be expressed in a definition.

(Pears/McGuinness)

The logical orderliness of propositions of colloquial language

5.5563 Alle Sätze unserer Umgangssprache sind tatsächlich, so wie sie sind, logisch vollkommen geordnet. – Jenes Einfachste, was wir hier angeben sollen, ist nicht ein Gleichnis der Wahrheit, sondern die volle Wahrheit selbst.

(Unsere Probleme sind nicht abstrakt, sondern vielleicht die konkretesten, die es gibt.)

5.5563 All propositions of our colloquial language are actually, just as they are, logically completely in order. That simple thing which we ought to give here is not a model of the truth but the complete truth itself.

(Our problems are not abstract but perhaps the most concrete that there are.)

(Ogden)

5.5563 In fact, all the propositions of our everyday language, just as they stand, are in perfect logical order. —That utterly simple thing, which we have to formulate here, is not a likeness of the truth, but the truth itself in its entirety.

(Our problems are not abstract, but perhaps the most concrete that there are.)

(Pears/McGuinness)

The requirement of the determinacy of meaning in the *Notebooks* (1915)

“Yes, this is the point: Can we just apply logic just as it stands, say in *Principia Mathematica,* straight away to *ordinary propositions*?

Of course we cannot disregard what is *expressed* in our propositions by means of endings, prefixes, changes of vowel, etc. etc.

But we do apply mathematics, and with the greatest success, to ordinary propositions, namely to those of physics. (...)

The difficulty is really this: even when we want to express a *completely definite* sense there is the possibility of failure. So it seems that we have, so to speak, no guarantee that our proposition is really a picture of reality.

The division of the body into *material points* as we have it in physics, is nothing more than analysis into *simple components*.

But could it be possible that the sentences in ordinary use have, as it were, only an incomplete sense (quite apart from their truth or false-hood), and that the propositions in physics, as it were, approach the stage where a proposition really has a complete sense?

When I say, "The book is lying on the table", does this really have a completely clear sense? (An EXTREMELY important question.) But the sense must be clear, for after all we mean *something* by the proposition, and as much as we *certainly* mean must surely be clear. (...)

Then are the propositions of physics and the propositions of ordinary bottom equally sharp, and does the difference consist only in life at the more consistent application of signs in the language of science?

Is it or is it not possible to talk of a proposition's having a more or less sharp sense?

It seems clear that what we mean must always be *"sharp"*.

Our expression of what we mean can in its turn only be right or wrong. And further the words can be applied consistently or inconsistently. There does not seem to be any other possibility.

(Wittgenstein, *Notebooks 1914-1916,* 20.6.15, 67e-68e)

The justifiability of the vagueness of ordinary phrases (1915)

It is then also clear to the *uncaptive* mind that the sense of the proposition "The watch is lying on the table" is more complicated than the proposition itself.

The conventions of our language are extraordinarily complicated. There is enormously much added in thought to each proposition and not said. (These conventions are exactly like Whitehead's 'Conventions'. They are definitions with *a certain generality of form.*) [*Cf.* 4.0002.]

I only want to justify the vagueness of ordinary sentences, for it *can* be justified.

It is clear that *I know* what I *mean* by the vague proposition.

Wittgenstein, *Notebooks 1914-1916,* 22.6.15, 69e-71e

The perfect orderliness of ordinary phrases in the *Philosophical Investigations* (1953)

"98. On the one hand, it is clear that every sentence in our language 'is in order as it is'. That is to say, we are not *striving* after an ideal, as if our ordinary vague sentences had not yet got a quite unexceptionable sense, and a perfect language still had to be constructed by us. - On the other hand, it seems clear that where there is sense, there must be a perfect order. —— So there must be perfect order even in the vaguest sentence."

"98. Einerseits ist klar, daß jeder Satz unsrer Sprache 'in Ordnung ist, wie er ist'. D. h., daß wir nicht ein Ideal *anstreben*: Als hätten unsere gewöhnlichen, vagen Sätze noch keinen ganz untadelhaften Sinn und eine vollkommene Sprache wäre von uns erst zu konstruieren. A Anderseits scheint es klar: Wo Sinn ist, muß vollkommene Ordnung sein. —— Also muß die vollkommene Ordnung auch im vagsten Satze stecken."

Criticisms addressed to Wittgenstein's approach of vagueness

Wittgenstein's paradoxical 'neglect' of the determinateness of sense (2004)

"Even the most casual readers of the *Philosophical Investigations* and the *Remarks on the Foundations of Mathematics* will appreciate that the author of those pages did not regard the sense of a statement as something that could be grasped a priori, with finality, as something in place of necessity, independently of the facts, and only subsequently applied to the contingent world. (note 1: The late Wittgenstein also leaves behind the other component of 'the determinateness of sense', namely the rejection of vagueness (e.g. PI§98)."

(Maddy, *The Logical Must*, p. 63, 2004)

Wittgenstein's rejection of vagueness (2012)

"My look at the *Tractatus* may, I hope, have clarified the sense in which Wittgenstein accuses his earlier self of having conceived (or rather "preconceived") of logic as having a "crystalline purity." Logic is the very structure of the world and imposes demands: No vagueness! Concepts must make sense in all possible situations! What is possible is absolutely fixed in advance! Hence what thinkers *can* meaningfully think is also fixed (and, as we saw, must be the same for all thinkers)! These are all demands that the later Wittgenstein comes to see as chimerical. I hear pathos in *Philosophical Investigations* when he asks: 'But what becomes of logic now? Its rigour seems to be giving way here.—But in that case doesn't logic altogether disappear? For how can logic lose its rigour? Of course not by our bargaining any of its rigour out of it.—The *preconceived idea* of crystalline purity can only be removed by turning our whole examination around. (One might say: the axis of reference of our examination must be rotated, but around the fixed point of our real need.)'"

(Putnam, *Philosophy in Age of Science*, pp. 349-350, 2012).

Russell's account of vagueness (1923)

"I propose to prove that all language is vague, and that therefore my language is vague, but I do not wish this conclusion to be one that you could derive without the help of the symbolism."

"One system of terms related in various ways is an accurate representation of another system of terms related in various other ways if there is a one-one relation of the terms of the one to the terms of the other, and likewise a one-one relation of the relations of the one to the relations of the other, such that, when two or more terms in the one system have a relation belonging to that system, the corresponding terms of the other system have the corresponding relation belonging to the other system. Maps, charts, photographs, catalogues, etc. all come within this definition in so far as they are accurate.

Per contra, a representation is vague when the relation of the representing system to the represented system is not one-one, but one-many. For example, a photograph which is so smudged that it might equally represent Brown or Jones or Robinson is vague. A small-scale map is usually vaguer than a large-scale map, because it does not show all the turns and twists of the roads, rivers, etc. so that various slightly different courses are compatible with the representation that it gives. Vagueness, clearly, is a matter of degree, depending upon the extent of the possible differences between different systems represented by the same representation. Accuracy, on the contrary, is an ideal limit."

"The fact that meaning is a one- many relation is the precise statement of the fact that all language is more or less vague."

"A great mistake to suppose that vague knowledge must be false. On the contrary, a vague belief has a much better chance of being true than a precise one, because there are more possible facts that would verify it. … we can distinguish between accuracy and precision. A belief is *precise* when only one fact would verify it; it is *accurate* when it is both precise and true."

Vagueness, 1923.